The Adaptive Rank Filtration

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1. Introduction

Rank filtering is a special kind of image processing. Its main advantages are stability of operation under high level of the noise and the absence of the edge distortions / 1, 2 /. Rank filtration is perhaps not investigated enough because of its limited flexibility. The purpose of this article is to introduce the way for increasing rank filters flexibility.

2. The Statement of the Task

There is the collection $G = \{g_{ij}\}, i=1,...,M_1, j=1,...,M_2$. It is necessary to find a new element g'_{ij} for each element $g_{ij} \in G$. The element g'_{ij} must satisfy these constraints:

- the element g^*_{ij} is defined by its neighborhood (kernel), i.e. $G_{ij} = \{g^{ij}_{vw}\}$, $G_{ij} \subset G$, $v = 1,...,N_1$, $w = 1,...,N_2$, for which this element is the central one;
- $g'_{ij} = f(G_{ij}) = g^{ij*}_{Rij}$ (here: R_{ij} - is the rank of the current neighborhood, $g^{ij}*$ – the element of regulating current neighborhood, which elements are related in increase.

If R_{ij} = ***const*** it is well-known rank filter and if $R_{ij} = INT(N_1N_2 / 2) = g^{ij*}_{Rij} = M_{ij}$ – median filter (the most popular subset of the rank filter).
The case when the rank R_{ij} depends on its neighborhood is a case of special interest:

$$R_{ij} = \psi(G_{ij}) \quad . \tag{1}$$

Such filters we'll call Adaptive Rank Filters (***ARF***).

3. The Principle of Adaptive Rank Filter Organization

Let's investigate several different principles of ***ARF*** organization and mark beforehand that the collection **G** we'll mean as one frame.
The kind of ***ARF*** and, consequently, the results of filtering are defined by the dependence (1) of the rank $\mathbf{R_{ij}}$ on the current neighborhood $\mathbf{G_{ij}}$. The optimum is the dependence of the rank $\mathbf{P_{ij}}(\mathbf{G_{ij}}) \geq 0$, which we'll call the power of current neighborhood, i.e.:

$$R_{ij} = \psi(P_{ij}(G_{ij})) \, .$$

One kind of the neighborhood dependence is:

$$P_{ij}(G_{ij}) = \sum_{v=1}^{N1}\sum_{w=1}^{N2} a_{vw}\, g_{vw}^{ij} + \sum_{v=1}^{N1-1}\sum_{w=1}^{N2-1} b_{vw}(|g_{vw}^{ij} - g_{v(w+1)}^{ij}| + |g_{vw}^{ij} - g_{(v+1)w}^{ij}| + |g_{vw}^{ij} - g_{(v+1)(w+1)}^{ij}|. \qquad (2)$$

Here coefficients $\boldsymbol{a}_{vw}$ and $\boldsymbol{b}_{vw}$ ($\boldsymbol{v} = 1,...,N_1$, $\boldsymbol{w} = 1,...,N_2$) are ***ARF*** parameters. In case, when $\boldsymbol{a}_{vw} = 1$ and $\boldsymbol{b}_{vw} = 0$ - the power of the neighborhood $\boldsymbol{P}_{ij}(\boldsymbol{G}_{ij})$ is proportional to the average of image brightness in the kernel. If $\boldsymbol{a}_{vw} = 0$ and $\boldsymbol{b}_{vw} = 1$ - the power of current neighborhood $\boldsymbol{P}_{ij}(\boldsymbol{G}_{ij})$ depend on derivative of image in current kernel.

4. The Modeling Results of ARF

A simulation model of ***ARF*** was created for investigation the ***ARF*** concept and the influence of different parameters on the results of filtration. Parameters of the simulation model are: input image, kernel size, and the collection of the coefficients $\mathbf{a}_{vw}$ and $\mathbf{b}_{vw}$ in the expression (2).
The modeling results giving below for the case, when the coefficients in the dependence (2) $\mathbf{a}_{vw} = 1$ and $\mathbf{b}_{vw} = 0$, i.e., the power of current neighborhood is defined by the expression:

$$P_{ij}(G_{ij}) = \sum_{v=1}^{N1}\sum_{w=1}^{N2} g_{vw}^{ij}.$$

The dependence of the rank to power of current neighborhood is the increasing function:

$$R_{ij} =]\frac{\varphi_1}{P_{ij}(G_{ij})}[\,.$$

Here the mark **] [** means the nearest integer of it's inside magnitude.
Use of this kind of the filter allows one to raise the image contrast. This effect is most marked when the function $\boldsymbol{\varphi}_1(\boldsymbol{P}_{ij}(\boldsymbol{G}_{ij}))$ is the following:

$$\boldsymbol{R}_{ij} = \begin{cases} 1 & -\,when\ \boldsymbol{P}_{ij} \le \boldsymbol{K}, \\ N_1 N_2 & -\,when\ \boldsymbol{K} < \boldsymbol{P}_{ij}; \end{cases}$$

$$1 < \boldsymbol{K} < N_1 N_2\,.$$

Here ***K*** - is a parameter of the filter that defines the threshold of brightness. The brightness is raised as the result of filtration when the brightness of the image element is more than this threshold. Otherwise, the brightness is decreased. Reduction of ***K***'s magnitude raises brightness average and vice versa (see figure 1 and 2).

Figure 1. Original image

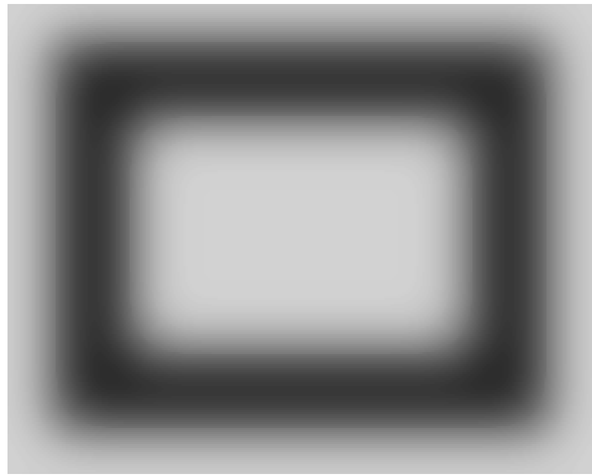

Figure 2. Result image (rank is proportional to the brightness)

It is possible to get the line finding of the same brightness gradient of the image by using ***ARF***, in which the dependence of the rank on the power of current neighborhood is the decreasing function:

$$R_{ij} = \left] \frac{\varphi_2}{P_{ij}(G_{ij})} \right[.$$

As in previous case the effect is most marked when the function $\boldsymbol{\varphi_2(P_{ij}(G_{ij}))}$ is the following:

$$\boldsymbol{R_{ij}} = \begin{cases} N_1N_2, & \text{when } \boldsymbol{P_{ij}} \le \boldsymbol{K}, \\ 1, & \text{when } \boldsymbol{K} < \boldsymbol{P_{ij}}; \end{cases}$$

$$1 < \boldsymbol{K} < N_1N_2 .$$

Here the parameter ***K*** defines the level of brightness gradient of the image above which there is line finding.

The superposition of two effects giving above by using two types of ***ARF*** is possible if the following of dependencies of the rank on the power are used:

$$\boldsymbol{R_{ij}} = \begin{cases} 1, & \text{when } \boldsymbol{P_{ij}} \le \boldsymbol{K_1}, \\ N_1N_2, & \text{when } \boldsymbol{K_1} < \boldsymbol{P_{ij}} \le \boldsymbol{K_2}, \\ 1, & \text{when } \boldsymbol{K_2} < \boldsymbol{P_{ij}}; \end{cases} \qquad \boldsymbol{R_{ij}} = \begin{cases} N_1N_2, & \text{when } \boldsymbol{P_{ij}} \le \boldsymbol{K_1}, \\ 1, & \text{when } \boldsymbol{K_1} < \boldsymbol{P_{ij}} \le \boldsymbol{K_2}, \\ N_1N_2, & \text{when } \boldsymbol{K_2} < \boldsymbol{P_{ij}}; \end{cases}$$

$$1 < \boldsymbol{K_1} < N_1N_2; \qquad 1 < \boldsymbol{K_2} < N_1N_2.$$

Here $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are the parameters of filter. These parameters influence on the work of the filter as in

the two previous cases.
So, the given filter can both raise the image contrast and finds line of same brightness gradient of initial image.

Let the dependence of the rank on the solidity in ***ARF*** be the following:

$$R_{ij} = \begin{cases} 1, & \text{when } P_{ij} \le K_1, \\ N_1N_2, & \text{when } K_1 < P_{ij} \le K_2, \\ 1, & \text{when } K_2 < P_{ij} \le K_3, \\ N_1N_2, & \text{when } K_3 < P_{ij}, \\ \quad \ldots; \end{cases} \qquad R_{ij} = \begin{cases} N_1N_2, & \text{when } P_{ij} \le K_1, \\ 1, & \text{when } K_1 < P_{ij} \le K_2, \\ N_1N_2, & \text{when } K_2 < P_{ij} \le K_3, \\ 1, & \text{when } K_3 < P_{ij}, \\ \quad \ldots; \end{cases} \qquad (3)$$

$$1 < K_1 < N_1N_2, \quad 1 < K_2 < N_1N_2, \quad 1 < K_3 < N_1N_2, \ldots .$$

The using of the filter with given dependence (3) allows both – raising the image contrast and finding line of same brightness gradient of initial image (see figure 3 and 4).

Figure 3. Original image

Figure 4. Result image (rank is inverse proportional to the brightness)

Unlike the previous case, the greater the brightness gradient of initial image is the – greater is the number of concentric lines appears on the output image. Moreover, its quantity does not exceed the number of negative changes (from N_1N_2 to 1) function (3).
It is necessary to the mark, that kernel's size should be big enough (N_1, $N_2 > 2Q$, Q - the number of negative changes pointed above) for observation of this effect.

ARF in all abovementioned cases decreases the noise just like the standard rank filter. The noise is decreased more in parts of the image, which have less brightness fluctuations.

5. Summary

The new generalized rank filters, have more functional possibilities than the usual rank filters ***ARF***, yet retain their main advantage - namely stability of operation under high levels of noise.

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6. Bibliography

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